Should Economists Listen to Educational Psychologists? An Economic Model of Teaching Practices and Student Motivation*

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Abstract

This paper focuses on teacher quality by studying teaching practices and their effects on student motivation and performance. We develop a model that builds on achievement goal theory in which a teacher regulates student motivation through the choice of teaching practices and the classroom environment. We show that a teacher who increases students' extrinsic motivation by emphasizing grades and the demonstration of ability is able to motivate high-ability students, at least in the short term. For students of low or medium ability, with risk aversion, avoidance behavior, or when taking into account that schooling is a long-term project, the teacher fosters higher achievements by designing classroom environment that is more centered on mastery and self-referenced standards. By doing so, the teacher develops students' intrinsic motivation, their capacity to overcome failure, and their long-term motivation for schooling. We also show that teachers' choice of classroom environment is influenced by the costs of the different teaching practices, by whether they are utilitarian or Rawlsian maximizers, and by their time preferences. We draw some policy implications from our findings.

- **Keywords:** Teaching Practices, Student Motivation, Student Effort, Student Achievement, Achievement Goal Theory
- **JEL Codes:** D03, I21

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1 Introduction

It is widely recognized that most schools experience gaps between their mandate, to promote high academic achievement by their students, and their performance. These gaps can to a large extent be explained by lack of student effort. According to educational psychologists, the primary determinant of student effort is their motivation (e.g., Wigfield, Eccles, Roeser and Schiefele, 2009). Educational psychologists, as opposed to most education economists, regard motivation as a multifaceted phenomenon. They see students as being affected by four motivational factors when performing a learning task: their extrinsic valuation of the task, their intrinsic interest, their self-concept of ability, and their perception of control. These factors evolve through students' stages of development and their ups and downs in terms of achievement. Moreover, educational psychologists argue that teachers can alter students' motivational patterns through the choice of teaching practices and the design of the classroom environment (Ames, 1992; Wolters, 2004).

In this article, we develop an economic model to explore how teachers take into account the role of student motivation and choose the teaching practices and the classroom environment that best match student needs. We emphasize that, since schooling is compulsory and a long-term contract, the teachers' role is to motivate students, keep them on track and prevent them from dropping out.[1] To specify the set of teaching practices and how they relate to student motivation, effort, and performance, we rely on the achievement goal theory (Nicholls, 1984; Dweck, 1986; Ames, 1992), which comes from educational psychology.

This theory emphasizes the dual role of classroom structures and achievement goals. Achievement goals can be understood as a student's subjective representation of the purposes of the task to perform, the way success is defined, and the role of effort and ability in achievement. The achievement goal literature considers three main goals (Elliot, 1999): Students with a mastery goal focus on learning, developing new skills, and improving their competence. They use self-referenced standards and view success as evidence of effort. Students with a performance goal focus on proving their competence to themselves and others; they want to obtain high grades or outperform other students. They believe in normative standards and that performance strongly depends on and therefore signals their ability. Students with an avoidance goal want to avoid exhibiting incompetence. A classroom structure is

[1] Students have no outside option before a certain age and the length of compulsory education is determined ex-ante. Teachers should therefore do their best to keep students on track. This situation is in sharp contrast to firms where workers have outside options and employers can fire inefficient employees. This justification for teachers regulating student motivation and effort also echoes Rosen (1987). He stresses that learning basic skills (e.g., compulsory education) requires direct intervention by teachers. In contrast, as students move upward in the educational hierarchy and education becomes optional, students become more independent and can better indulge in self-regulation.

defined by the way a teacher designs tasks, shares authority and evaluates students' performance. The fundamental idea is that the choice of the classroom structure conveys a message to students about the learning goals and alters their initial goal orientation (Ames, 1992). A mastery goal structure refers to teaching practices that emphasize understanding and personal improvement. A performance goal structure refers to teaching practices that emphasize grades and rankings.[2] When students perceive a classroom structure as being focused on mastery (performance), they are likely to become more mastery-goal-oriented (performance-goal-oriented) (Wolters, 2004).

Extensive empirical research in the field of education psychology has shown that a mastery goal fosters adaptive study behaviors such as effort, deep processing of the learning material, task enjoyment, low levels of test anxiety, and persistence in the face of difficulties or failure (Anderman and Wolters, 2006). There is a noteworthy omission from the list of positive outcomes, however: empirical studies have not established a link between mastery and academic achievement. This puzzle has been explained by exams consisting of multiple choice questionnaires, which may favor surface learning over deep learning. Furthermore, mastery-oriented students seem to spend time on material that is personally interesting to them, but not relevant to the test (Senko and Miles, 2008).

In contrast, empirical research shows that a performance goal fosters effort and academic achievement (Anderman and Wolters, 2006). The positive relationship between performance goals and grades has been explained by the fact that performance-oriented students seek to align their learning agenda with that of the teacher by carefully trying to identify the assessment criteria (Senko and Miles, 2008). However, there are also negative consequences associated with a performance goal. Performance-oriented students tend to have higher levels of test anxiety. They are also more inclined to use avoidance strategies (such as self-handicapping and not seeking help) to protect their self-worth by deflecting attention from ability. Several researchers have also suggested that it could be more difficult for performance-oriented students to preserve their level of engagement in the long run, notably after a failure (e.g., Covington and Omelich, 1979).

In this paper, we develop a theoretical framework to understand how the classroom structure affects students' motivation, effort, and performance. This framework enables us to describe how teachers' characteristics (such as their teaching costs, objective function, and time preferences) and students' characteristics (such as their initial goal orientation, ability, degree of risk aversion, and time preferences) intervene in the choice of optimal classroom structure. We consider a situation with a teacher and a class of students with a learning task to achieve. There is a test at the end of the

[2] We will describe the practices more precisely later.

period to verify whether knowledge has been acquired. The teacher chooses a classroom structure, which alters both the students' initial goal orientation, i.e. their motivational pattern to exert effort, and the efficiency of the effort exerted. The teacher's objective is to maximize student grades. The initial framework is static: the teacher and students only meet for one period. We later consider a dynamic framework for studying the management of student motivation in the long run.

We first consider a classroom with homogeneous students. We show that, when faced with high-ability students, the teacher can promote academic achievement by designing a performance goal structure (Proposition 1). The reason is twofold. First, the effort exerted by a student under this classroom structure reflects his ability level. Second, students are more focused on the teacher's demands, so that each unit of effort exerted is more efficient. Faced with low- or intermediate-ability students, the teacher cannot rely solely on extrinsic motivation to foster performance. For such students, the teacher promotes better performances by choosing a more mastery-oriented classroom structure. By emphasizing the importance of mastery, the teacher creates a learning environment in which students' exertion of effort is fueled by their interest in the task, and where they obtain some satisfaction independently of the test result. This has two consequences. First, students display an effort level that is independent of their ability level (Proposition 1). Second, students are much less affected by cognitive biases, such as avoidance behavior and test anxiety, which could destroy their motivation to exert effort (Propositions 2 and 3).

When extending the model to include heterogeneous students, the choice of the optimal classroom structure will also be affected by the objective being pursued by the teacher. Notably, we show that, when students' abilities are more dispersed, a Rawlsian teacher chooses a classroom structure that is more oriented toward mastery in order to keep at-risk students motivated. A utilitarian teacher, on the contrary, chooses a classroom structure that is more oriented toward performance in order to maintain the average student performance (Proposition 4).

The results in Proposition 1-4 were obtained in a static framework. To take into account that schooling is a long-term contract and that building motivation is a long-term process, we consider a dynamic version of the framework. We show that the teacher, if sufficiently patient, chooses a first-period classroom structure that is more mastery-oriented than in the static case. In doing so, the teacher develops a long-term failure tolerance among students, even if this is at the expense of a (slight) short-run decrease in performance (Proposition 5).

This article is related to both the literature on teacher quality that considers teachers as the schools' most important asset, and the new microeconomics of education, which considers student effort to be the most important input to education production. This latter literature (e.g., Correa and Gruver, 1987; Bishop, 1994; Costrell, 1994; Betts, 1998; Akerlof and Kranton, 2002; Bonesrønning, 2004; De Fraja and Landras, 2006; De Fraja, Oliveira, and Zanchi, 2010; Fryer, 2010; Bettinger, 2012) either focuses on students' level of effort in a prisoner's dilemma situation, the use of extrinsic motivation (e.g., educational standards, grading practices, competition, monetary incentives), or students' social position and identity in schools. Students' intrinsic sources of motivation, and the efficiency and the dynamics of effort are often neglected. The teacher's role in avoiding a low effort-low effort equilibrium characterizing a prisoner's dilemma situation is also often neglected.

The vast empirical literature on teacher quality commonly measures quality in terms of pre-service characteristics, teacher experience, and teacher value-added (e.g., Hanushek and Rivkin, 2006). Only a few papers study teaching practices and how they relate to student outcomes. Algan, Cahuc and Shleifer (2013) analyze the relationship between teaching practices and social capital. They study horizontal teaching practices (working in groups) versus vertical teaching practices (teachers lecturing). They find that the former generates social capital. Their paper is therefore more related to the growing literature on non-cognitive skills and how such assets relate to economic payoffs. Rouse, Hannaway, Goldhaber, and Figlio (2013) show how instructional practices changed in meaningful ways due to accountability pressure. However, they study a reduced-form effect, and neglect the mechanisms relating the changes in teaching practices to student effort and achievement.

Our paper is one of few to study teacher quality through the lens of teaching practices and student motivation. We first study the management of motivation in the short run: for given student characteristics, how can the teacher enhance student motivation, effort and performance? How can the teacher take into account cognitive biases such as students withdrawing from challenging tasks and students suffering from text anxiety? We also study how the teacher's choice of classroom structure depends on the relative costs associated with the different structures and on whether the teacher is a utilitarian or Rawlsian maximizer. We next study the management of motivation in the long run: how can the teacher design a classroom environment to maintain students' motivation over time, particularly after failure? The article proceeds as follows: Section 2 presents the static model. Section 3 presents the dynamic framework. Section 4 draws some policy implications from the model. Section 5 concludes.

2 Static Management of Student Motivation

In this section, we consider a model with a teacher and a class of students who interact during one period. In section 2.1, we consider a class composed of homogeneous students. We first study the case where students have two initial goals, mastery and performance, and characterize the equilibria. Thereafter, we extend the analysis to include an avoidance goal. Finally, we introduce risk aversion in student preferences to incorporate the idea that students may suffer from test anxiety. In section 2.2, we consider a class composed of heterogeneous students and study how the choice of classroom structure depends on whether the teacher is a utilitarian or a Rawlsian maximizer.

2.1 The case of homogeneous students

2.1.1 The Model

Since the class is composed of identical students, we can focus on the problem of one (representative) student.

The student ("he"). He has knowledge to acquire. There is a test at the end of the period to verify whether this knowledge has been acquired or not. The student can either pass or fail the test. We assume that the test result is a perfect indicator of the acquisition of knowledge. The student is endowed with an initial goal orientation $\gamma \in [0, 2]$, (cognitive) ability $\theta \in [0, 1]$, and he exerts effort $e \in [0, 1]$. The probability of passing the test is increasing with ability and effort. We denote x as the random variable equal to 1 when the student is successful and 0 otherwise.

The teacher ("she"). We assume that she has complete information about the representative student's characteristics.[3] She chooses a classroom structure, s, in a continuum of differentiated structures, $[0, 1]$. The structure describes the way the teacher designs learning tasks, shares authority and evaluates students' performance. The structure $s = 0$ corresponds to the situation in which the teacher chooses a pure mastery goal structure.[4] The structure $s = 1$ corresponds to the situation in which the teacher chooses a pure performance goal structure.[5] An intermediate structure, $s \in (0, 1)$,

[3] This is a fair assumption faced with a representative student. Later, we will consider heterogeneous students and the teacher will only know the distribution of student characteristics.

[4] Under a pure mastery goal structure, the teacher uses differentiated learning tasks to challenge all students. Students are involved in the decision making and are given opportunities to participate during the class. The time available to perform a task is flexible and the recognition of achievement is private. The teacher emphasizes effort. The primary objective consists of mastery of the learning material and self-referenced improvements (Ames, 1992).

[5] Under a pure performance classroom structure, the teacher exerts total control over the classroom activities and provides whole class instruction. Learning tasks are repetitive and the time available to perform a particular task is

is referred to as a multiple goal structure, and corresponds to a mixture of the two pure classroom structures. For example, we can think of the situation where the teacher spends a proportion $1-s$ of the time on teaching practices emphasizing mastery, and a proportion s on teaching practices emphasizing performance.[6] The choice of s alters the student's initial goal orientation and the probability of succeeding in the test.

Test result. We assume that it is equal to

$$x = \begin{cases} 1 & \text{with probability} \quad \theta s e \\ 0 & \text{with probability} \quad 1-\theta s e \end{cases} \tag{1}$$

The probability of passing the test is increasing in the student's effort, e. It is also increasing in θs, which we interpret as the efficiency of effort. The higher the ability, the higher the efficiency of effort. Furthermore, we assume that, when the teacher stresses performance by choosing a higher s, the student becomes more attentive to her demands, so that the efficiency of effort and the probability of passing the test increase. This latter assumption is consistent with empirical observations (e.g., Senko and Miles, 2008).

Payoffs. The student maximizes the following expected utility function:

$$(\theta s e) \times u(1 + \gamma(1-s)e - 0.5e^2) + (1-\theta s e) \times u(\gamma(1-s)e - 0.5e^2) \tag{2}$$

with $u' > 0$ and $u'' \leq 0$. The student has two sources of utility, extrinsic and intrinsic, which correspond to the two achievement goals. The first term in $u(.)$ represents the payoff when pursuing a performance goal: when the student succeeds in the test, he is able to demonstrate his competence and he gets a positive payoff normalized to 1; when he fails the test, he gets 0. The term $\gamma(1-s)e$ represents the payoff for pursuing a mastery goal. It corresponds to the intrinsic satisfaction derived from acquiring knowledge, developing new skills and achieving a sense of mastery based on self-referenced standards, and appears irrespective of the test result. In fact, for a student with a mastery goal, the motivation for effort is not to get an A, for example in Spanish, but rather to speak Spanish fluently. The parameter $\gamma \in [0, 2]$ describes the *relative* importance of mastery goals versus performance goals in the student's initial goal orientation. We will refer to γ as the mastery-performance goal index.

fixed. The recognition of achievement is public. The teacher emphasizes that the primary objective of students is to obtain a good grade and to prove their ability (Ames, 1992).

[6] The dimensions defining the structures are referred to by the acronym TARGET: Task, Autonomy, Recognition, Grouping, Evaluation and Time (Ames, 1992). We model these different dimensions through the variable s.

This index will play an important role in the analysis because it defines the motivational pattern of the student. When γ is equal to 0, performance is the student's predominant initial goal orientation. When γ is equal to 2, mastery is the predominant initial goal orientation. We will show that, in this case, the student exerts the maximum level of effort even when his ability is nil (that is, when the probability of failing the test is equal to one). When γ is equal to 1, the student initially has a balanced motivational pattern. For the moment, we assume that γ does not depend on θ, which means that the goal orientation is not affected by the ability level. However, we will later extend the analysis to include a class of heterogeneous students. That will enable us to consider positive or negative correlations between θ and γ. Note that $\gamma(1-s)e$ is increasing in effort: in contrast to performance-oriented students who view success as evidence of ability, mastery-oriented students view success as evidence of effort (Ames, 1992). Furthermore, $\gamma(1-s)e$ is increasing when s decreases: the student's intrinsic satisfaction is higher as the teacher increases the mastery goal structure. Finally, the term $0.5e^2$ is the cost of exerting effort.

The teacher is risk neutral. Student achievement is the teacher's primary concern: she achieves a gross utility equal to w when the student succeeds in the test. We also take into account a possible cost difference between structures. Indeed, it is reasonable to think that mastery-oriented classroom structures require more involvement, attention, and understanding of student needs, and therefore more effort from the teacher.[7] We write the teacher's expected utility function as $(\theta se)w-(1-s)c$. When c is positive, the teaching cost is higher as the teacher increases the mastery classroom structure.

Timing of the game

- First, the teacher chooses a classroom structure, $s \in [0,1]$.
- Second, the student observes s and exerts effort $e \in [0,1]$.
- Third, the student takes the test and obtains a result $x \in \{0,1\}$.

2.1.2 The benchmark case

The strategies are s for the teacher and $e(s)$ for the student. We next characterize the subgame perfect equilibrium of the benchmark case (a risk-neutral representative student with two achievement goals). Risk neutrality corresponds to the case where $u(z)=z$. We consider the problem of the representative

[7] With a performance goal structure, the teacher gives whole class instruction, whereas with a mastery goal structure, teaching is more personalized and adapted to each student's needs.

student for a given classroom structure s. Maximizing expression (2) with respect to e yields

$$e^*(s) = \min\{\gamma + (\theta - \gamma)s, 1\} \tag{3}$$

When $\theta > \gamma$, the student is characterized by high ability and/or a low mastery-performance goal index (i.e., his extrinsic interest in the task is relatively more pronounced than his intrinsic interest). In this case, effort increases as the teacher chooses a more performance-oriented classroom structure (that is, a higher s). When $\theta < \gamma$, effort increases as the teacher chooses a more mastery-oriented classroom structure (that is, a lower s).

We now consider the maximization problem of the teacher. We first study the case where there is no cost difference between classroom structures: $c = 0$. The teacher solves $s^* = \arg\max_{s\in[0,1]}(\theta s e^*(s))w$. The solution is

$$s^* = \begin{cases} \frac{1}{2}\frac{\gamma}{\gamma-\theta} & \text{if} \quad \theta \leq \gamma/2 \\ 1 & \text{if} \quad \theta \geq \gamma/2 \end{cases} \tag{4}$$

The equilibrium is fully described by expressions (3) and (4). At equilibrium, the student's effort, $e^*(s^*)$, is equal to $\gamma/2$ when $\theta \leq \gamma/2$ and equal to θ when $\theta \geq \gamma/2$. We sum up the result in the following proposition:

Proposition 1 *At equilibrium, the teacher chooses a performance goal structure when $\theta \geq \gamma/2$. The student then exerts an effort proportional to his ability level. When $\theta < \gamma/2$, the teacher chooses a multiple goal structure. The student then exerts effort independently of his ability level.*

When the ability θ is higher than the mastery-performance goal index γ, both effort $e^*(s)$ and its efficiency θs increase as the teacher chooses a more performance-oriented classroom structure. Therefore the teacher chooses a pure performance goal structure, $s^* = 1$. When θ is between $\gamma/2$ and γ, the teacher still chooses $s^* = 1$. The student's effort would be higher if the classroom structure were more oriented toward mastery goals. However, this mastery-induced effort would be less efficient and the probability of passing the test would decrease. When θ is below $\gamma/2$, the teacher chooses a multiple goal structure that conveys both performance and mastery goals: $s^* = \gamma/(2\gamma - 2\theta)$.[8] In doing so, she induces an effort level $e^*(s^*) = \gamma/2$, that is independent of the student's ability θ. In this case, the benefit of breaking the ability-effort connection is higher than the loss resulting from the reduced efficiency of effort. Note, that while a low-ability student is motivated to exert effort under

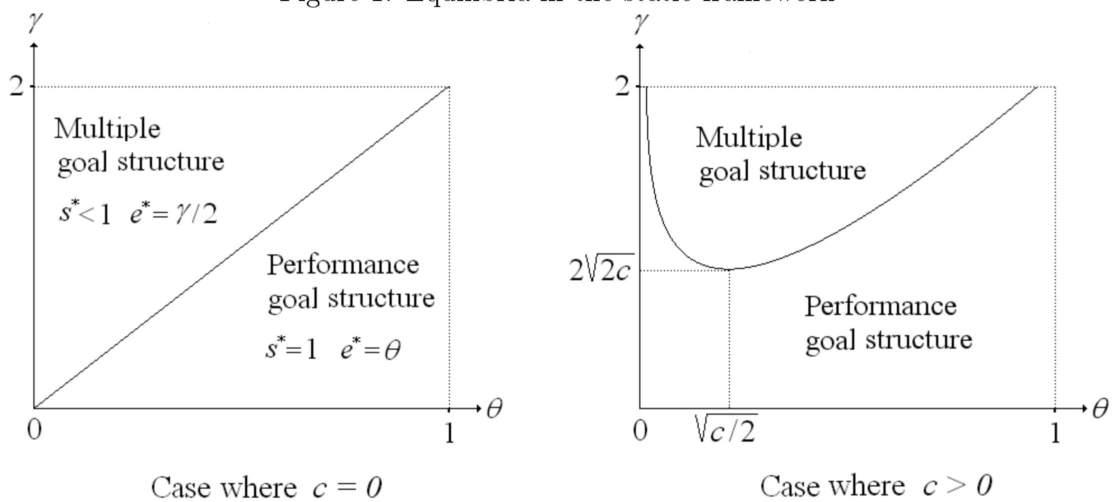
[8]Note that the optimal structure s^* is always larger than 1/2: a pure mastery goal structure would, for example, nullify the efficiency of the effort.

Table 1: Equilibrium Payoffs

	s^*	$e^*(s^*)$	U^{t^*}	U^{p^*}
If $\theta \geq \gamma/2$	1	θ	θ^2	$\frac{\theta^2}{2}$
If $\theta \leq \gamma/2$	$\frac{1}{2}\frac{\gamma}{\gamma-\theta}$	$\frac{\gamma}{2}$	$\frac{1}{4}\frac{\gamma^2\theta}{\gamma-\theta}$	$\frac{\gamma^2}{8}$

a multiple goal structure (at least if γ is not too small), he will hardly succeed in the test because of the role played by ability in the achievement probability function, θse. However, his achievement would be much worse under a pure performance structure. The equilibria are represented on the left side of Figure 1. The equilibrium path and payoffs are described in Table 1 (U^t denotes the teacher's payoff and U^p denotes the student's payoff).

Figure 1: Equilibria in the static framework



Some educational psychologists have long advocated the development of a multiple goal structure to foster students' achievement. For example, Linnenbrink (2005) studies the effects of mastery-, performance-, and multiple goal structures on middle school students' motivation, emotional well-being, help seeking, and achievement. The study shows that the classroom goal structure has a significant effect on help seeking and achievement and that the multiple goal structure has the most beneficial impact. According to educational psychologists, the multiple goal structure enables students to combine the best features from mastery and performance goals: empirical studies show that whereas mastery fosters interest, performance fosters students to become more attentive to the teacher's demands (Senko and Miles, 2008; Harackiewicz, Durik, Barron, Linnenbrink, and Tauer, 2008). Our framework helps to clarify these findings: When ability is low or intermediate, effort and its efficiency vary in opposite directions as the classroom structure changes. The multiple goal structure corresponds to the best mix of incentives and makes the student's effort independent of his ability. Motivating effort through the student's interest for the task is a first step toward achievement, but achievement also requires that the student stays focused on the teacher's demands. When ability is high, a performance structure causes the student to exert high and efficient effort. Thereby promoting high academic achievement.

If we consider a cost difference between structures, the teacher's objective function then reads $(\theta se)w-(1-s)c$, where $c > 0$. As explained above, designing a mastery-oriented structure requires more involvement from the teacher. Without loss of generality, we can take $w = 1$. Furthermore, let us assume that $c \leq \gamma^2/8$. Solving $\max_{s\in[0,1]} \theta se^*(s) - c(1-s)$, where $e^*(s)$ is given by (3), yields

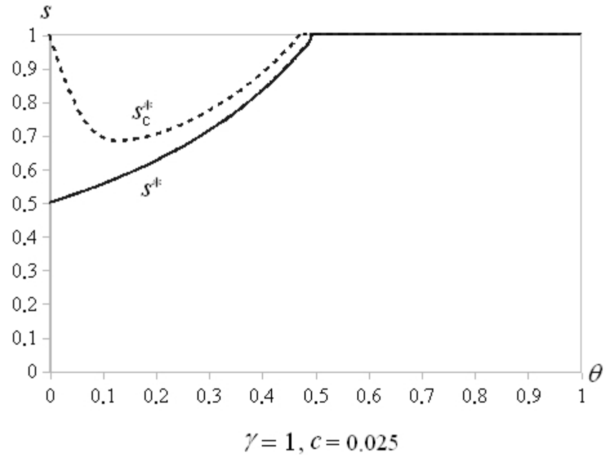
$$s_c^* = \begin{cases} 1 & \text{if} \quad \theta \leq \frac{\gamma-\sqrt{\gamma^2-8c}}{4} \\ \frac{c/\theta+\gamma}{2(\gamma-\theta)} & \text{if} \quad \frac{\gamma-\sqrt{\gamma^2-8c}}{4} \leq \theta \leq \frac{\gamma+\sqrt{\gamma^2-8c}}{4} \\ 1 & \text{if} \quad \theta \geq \frac{\gamma+\sqrt{\gamma^2-8c}}{4} \end{cases}$$

The equilibria are represented on the right side of Figure 1 and the optimal structure in Figure 2 for the balanced motivational pattern $\gamma = 1$ and the cost difference $c = 0.025$. Not surprisingly, the teacher chooses a more performance-oriented structure when there is a cost difference between structures: $s_c^* \geq s^*$. More interestingly, a performance goal structure is the optimal policy for (very) low-ability students. Designing a multiple goal structure would induce a higher effort from these students. Yet from the teacher's point of view, the increase in performance would not compensate for the increase in cost resulting from the multiple structure. Hence, the teacher is better off promoting a performance goal structure. Later, in section 4, we will draw some policy implications based on these findings. In the rest of the paper, we consider the case where there is no cost difference between classroom structures: $c = 0$. In this case, the teacher simply maximizes the expected grade of the student.

2.1.3 Introducing an avoidance goal

We now introduce the third achievement goal, avoidance. Students with an avoidance goal want to avoid exhibiting incompetence to protect their self-worth (Elliot, 1999). They use different strategies not to feel responsible in the case of failure. For example, they can choose to withdraw from effort and not participate in classroom activities since greater shame at failure is experienced under high-effort conditions (Covington and Omelich, 1985). They can also avoid asking for help and use self-

Figure 2: Classroom structures for different costs



handicapping strategies, such as partying the night before an exam. To model the avoidance goal, we assume that the student only agrees to participate in classroom activities (that is, to exert a positive level of effort) if he obtains an utility at least equal to $\underline{U}^p$. The level $\underline{U}^p$ represents the utility of adopting an avoidance goal and "failing with honor".[9]

To avoid taxonomy, we take $\gamma \leq 1$ and assume that $\gamma^2/8 < \underline{U}^p \leq \gamma^2/2$.[10] For a given classroom structure s, the expression (3) of student effort becomes $e^*(s) = \theta s + \gamma(1-s)$ and the associated utility is $U^p = 0.5(\theta s + \gamma(1-s))^2$. The teacher maximizes the expectation of the grade, $\theta s e^*(s)$, subject to the participation constraint $U^p \geq \underline{U}^p$.

If $\theta \leq \gamma/2$, the utility for the student in the unconstrained situation, $\gamma^2/8$, is below the reservation utility, $\underline{U}^p$. Therefore, the teacher needs to align her preferences with the student's in order to prevent avoidance. To do so, she reduces s^* from the unconstrained level, $\frac{1}{2}\frac{\gamma}{\gamma-\theta}$, down to the level s_p^* satisfying $0.5(\theta s^* + \gamma(1-s^*))^2 = \underline{U}^p$, that is, $s_p^* = \frac{\gamma - \sqrt{2\underline{U}^p}}{\gamma-\theta}$. If $\gamma/2 \leq \theta \leq \gamma$, the teacher reduces s^* from the unconstrained level, 1, down to the level $s_p^* = \min\left\{1, \frac{\gamma-\sqrt{2\underline{U}^p}}{\gamma-\theta}\right\}$. If $\theta \geq \gamma$, the teacher chooses $s_p^* = 1$. In this case, the associated student utility, $\theta^2/2$, is larger than $\gamma^2/2$ and the participation constraint is satisfied. The results are summarized in the following proposition.

Proposition 2 *To prevent the student from adopting an avoidance goal, the teacher increases the mastery goal structure when the student's ability is low or intermediate.*

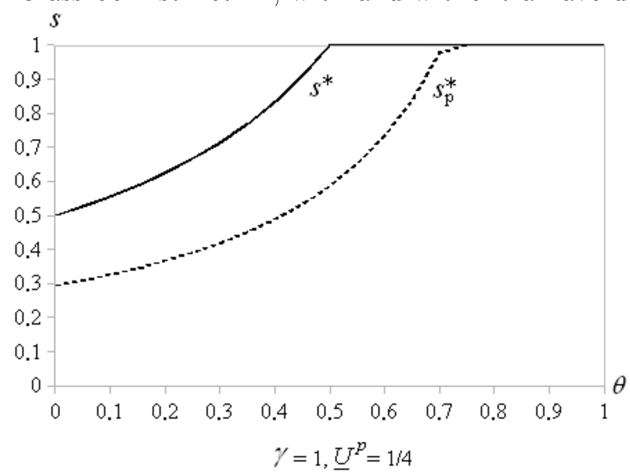
We compare the classroom structure with and without the avoidance goal in Figure 3 for $\gamma = 1$

[9]The achievement goal theory sometimes distinguishes between performance avoidance (avoiding normative incompetence) and mastery avoidance (avoiding intrapersonal incompetence), e.g., Hadsel (2010). We follow Elliot (1999), who does not differentiate between these two types of avoidance.

[10]Condition $\underline{U}^p > \gamma^2/8$ guarantees that the reservation utility is sufficiently high, so that the participation constraint will come into play in some cases. Condition $\underline{U}^p \leq \gamma^2/2$ guarantees that avoidance is not too desirable for the student.

and $\underline{U}^p = 1/4$. In our framework, a student is more inclined to pursue an avoidance goal if his ability is low or intermediate. In this case, the teacher can develop the student's intrinsic motivation by increasing the mastery goal structure and thereby preventing him away from adopting the avoidance goal. Our results are consistent with empirical findings from the achievement goal literature: mastery structures (performance structures) tend to be negatively (positively) associated with avoidance and self-handicapping behaviors (Turner, Midgley, Meyer, Gheen, Anderman, Kang, and Patrick, 2002).

Figure 3: Classroom structure, with and without an avoidance goal



2.1.4 Test anxiety, locus of control, and risk aversion

We now assume that the student is risk averse. Risk aversion in a learning context might be related to test anxiety: a cognitive bias associated with a certain level of discomfort before and during taking a test. Test anxiety can drastically impede students' ability to perform well on tests and it can negatively affect their feelings about themselves and schooling. Alternatively, risk aversion can be linked to the concept of locus of control, that is, the extent to which students attribute success or failure to internal or external factors. Alternatively, risk aversion can be linked to the concept of locus of control, that is, the extent to which students attribute success or failure to internal or external factors. Whereas "internals" believe that success is primarily the result of their own behavior, "externals" believe that success is the result of chance or the teacher's actions (Findley and Cooper, 1983).[11]

[11] Risk aversion in education is normally associated with the risk-return trade-off related to investment in educational attainment (e.g., Christensen, Joensen and Nielsen, 2007; Kuehn and Landeras, 2013). Here, we focus instead on the effort-performance risk associated with classroom participation and test-taking.

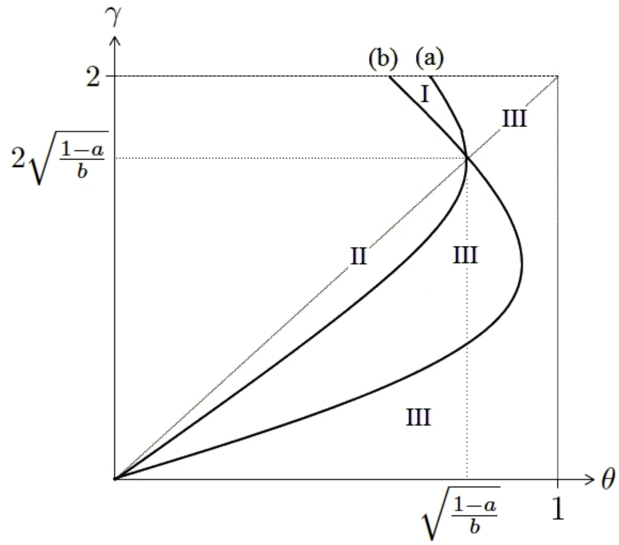
To keep tractability, we consider the case where the representative student has a constant absolute risk aversion, $r > 0$, by taking $u(z) = -exp(-rz)$ in expression (2). Maximizing the expected utility for the student yields

$$e_r^*(s) = \min\left\{\frac{1 + b\theta\gamma(1-s)s - \sqrt{\Delta}}{2b\theta s}, 1\right\} \tag{5}$$

where $\Delta = (1-b\theta\gamma(1-s)s)^2 - 4ab\theta^2 s^2$, $a = [1-exp(-r)]/r$ and $b = 1-exp(-r)$. When comparing the effort level under risk aversion, $e_r^*(s)$, to the effort level under risk neutrality, $e^*(s)$ given by expression (3), it is possible to verify that, for a given classroom structure s, we have $e_r^*(s) > e^*(s)$ if and only if $\theta > \left((s-1)\gamma + \sqrt{(s-1)^2\gamma^2 + 4(1-a)/b}\right)/2s$. Therefore, high-ability students exert more effort under risk aversion in order to reduce the uncertainty associated with the test. Conversely, students with low ability exert less effort under risk aversion. Exerting less effort allows these students to reduce the uncertainty associated with the test, even if it is at the expense of decreasing the probability of succeeding.

We now solve the maximization problem of the teacher, $\max_{s\in[0,1]}(\theta e_r^*(s)s)w$. The first order condition of the program is $2b\gamma\theta(1-2s)\sqrt{\Delta} - \Delta' = 0$ where Δ' is the derivative of Δ with respect to s. The first order condition has a closed form solution s_r^*, but its expression is excessively complicated and hence not displayed. It is more interesting to compare the choice of classroom structure with and without risk aversion in Figure 4. Working with the first-order condition of the program gives the two conditions represented in the figure: $s_r^* \leq 1$ if $\theta \leq \frac{2\gamma}{4a+b\gamma^2}$ (curve (a)) and $s_r^* \leq \frac{1}{2}\frac{\gamma}{\gamma-\theta}$ if $\theta \leq \frac{4\gamma(1-a)}{4(1-a)+\gamma^2 b}$ (curve (b)). In zone I, we have $s_r^* > s^*$, where s^* is given by (4). Faced with a risk-averse student, the teacher chooses a classroom structure that is more oriented toward performance goals if the student has a high level of ability and a high mastery-performance goal index. Putting more pressure on this type of student is efficient because they then exert more effort to better control the test result. In zone II, we have $s_r^* < s^*$. Faced with a risk-averse student, the teacher chooses a structure that is more oriented toward mastery if ability is low or intermediate. In doing so, the teacher develops the student's intrinsic motivation, reduces the uncertainty associated with the test, and elicits more effort. In a sense, the multiple structure affects the student's locus of control and makes him more internal: for a mastery-oriented student, success is primarily viewed as mastering a task (something over which the student has control), but less as obtaining a good grade (something over which the student has less control). In zone III, we have $s_r^* = s^* = 1$. We sum up the main results in the following proposition.

Figure 4: Classroom Structures with a Risk Averse Student



Proposition 3 *Faced with a risk-averse student, the teacher increases the performance goal structure if the student has a high level of ability and a high mastery-performance goal index, whereas, for a student with a low or intermediate level of ability, the teacher increases the mastery goal structure.*

Our results are related to the literature on self-protection in which a risk-adverse agent can increase the probability of success by carrying out a costly self-protection activity. Contrary to what intuition suggests, a more risk-adverse agent does not necessarily exert more effort since exerting less effort is a way of reducing the uncertainty (Dionne and Eeckhoudt 1985; Jullien, Salanié and Salanié; 1999).

In a learning context, Skinner, Zimmer-Gembeck, and Connell (1998) show that children who believe teachers are supportive and care about their progress develop a more positive sense of control over their outcomes. They are less anxious and perform better academically. This is related to our results, since teachers should favor a mastery goal structure when faced with students who are risk averse, but only when students have a low or intermediate level of ability.

2.2 The case of heterogeneous students

We now extend the analysis beyond the case of a representative student by considering the situation where the teacher faces a heterogeneous group of students. This extension is interesting for two reasons: First, how does the teacher adapt her choice of the optimal classroom structure when faced with students of different abilities and different motivational patterns? Second, how does the teacher behave in cases where ability and the mastery-performance goal index are positively or negatively correlated?

To answer these questions, we consider a class with a population of risk-neutral students of size one. Each student is characterized by his ability θ and his mastery-performance goal index, γ. The parameters θ and γ are distributed according to the density $f(\theta,\gamma)$ defined on $[\underline{\theta},1]\times\left[\underline{\gamma},1\right]$ with $\underline{\theta}>0$ and $\underline{\gamma}>0$. We assume that $f(\underline{\theta},\underline{\gamma})>0$. Let ρ denote the coefficient of correlation between θ and γ. We consider two types of teachers who differ in the objective they pursue: The utilitarian teacher maximizes the average expected grade in the class, $\int_0^1\int_0^1 \theta s e^*(\theta,\gamma,s)f(\theta,\gamma)d\gamma d\theta$ (where $e^*(\theta,\gamma,s)=\theta s+\gamma(1-s)$). The optimal classroom structure is[12]

$$s^*_{ut}=\begin{cases}\frac{1}{2}\frac{E[\gamma]+\rho\sigma(\theta)\sigma(\gamma)/E[\theta]}{E[\gamma]-E[\theta]-\sigma^2(\theta)/E[\theta]+\rho\sigma(\theta)\sigma(\gamma)/E[\theta]} & \text{if} \quad E\left[\theta\right]+\frac{\sigma^2(\theta)}{E[\theta]}\leq\frac{1}{2}\left(E\left[\gamma\right]+\frac{\rho\sigma(\theta)\sigma(\gamma)}{E[\theta]}\right)\\ 1 & \text{if} \quad E\left[\theta\right]+\frac{\sigma^2(\theta)}{E[\theta]}\geq\frac{1}{2}\left(E\left[\gamma\right]+\frac{\rho\sigma(\theta)\sigma(\gamma)}{E[\theta]}\right)\end{cases}$$

The Rawlsian teacher maximizes the expected grade of the most at-risk student $(\underline{\theta},\underline{\gamma})$, that is, the student with the lowest ability and lowest intrinsic motivation. We can use the results from section 2.1.2 to determine the optimal structure. The Rawlsian teacher chooses the pure performance goal structure $s^*_{ra}=1$ if $\underline{\theta}\geq\underline{\gamma}/2$, but the multiple goal structure $s^*_{ra}=\underline{\gamma}/(2\underline{\gamma}-2\underline{\theta})$ if $\underline{\theta}<\underline{\gamma}/2$.

To analyze how the two types of teacher adapt the classroom structure when faced with heterogeneous students and to compare their behavior, it is convenient to define the concept of "mean-preserving class diversification". Suppose that the class was homogeneous before becoming heterogeneous. We say that the class has undergone a mean-preserving diversification if the average ability and the average mastery-performance goal index are equal in the initial state and the final state. More precisely,

Definition. Suppose that the density was $\tilde{f}(\theta,\gamma)$ before being $f(\theta,\gamma)$. We say that this change corresponds to a mean-preserving class diversification if $\tilde{E}\left[\theta\right]=E\left[\theta\right]$, $\tilde{E}\left[\gamma\right]=E\left[\gamma\right]$, $\tilde{\sigma}(\theta)=\tilde{\sigma}(\gamma)=0$, and $\sigma(\theta)\geq0$ and $\sigma(\gamma)\geq0$ with at least one strict inequality.

Let s^* be the optimal classroom structure before the diversification. Note that the utilitarian and Rawlsian teachers choose the same structure in the initial state because the class is homogeneous. For the sake of simplicity, we focus on the case where the initial structure s^*, and the final structures, s^*_{ut} and s^*_{ra}, are interior solutions.[13] This yields the following result.

Proposition 4 *Consider a class that is subject to a mean-preserving diversification. Then the utilitarian teacher (respectively the Rawlsian teacher) chooses to increase the mastery goal structure*

[12]We use the fact that $\int_0^1\int_0^1 \theta s e^*(\theta,\gamma,s)f(\theta,\gamma)d\gamma d\theta = s^2E\left[\theta^2\right]+(1-s)sE\left[\theta\gamma\right] = s^2\left(E^2\left[\theta\right]+\sigma^2(\theta)\right)+(1-s)s\left(E\left[\theta\right]E\left[\gamma\right]+\rho\sigma(\theta)\sigma(\gamma)\right)$.

[13]That is, we have $E\left[\theta\right]<\frac{1}{2}E\left[\gamma\right]$, $E\left[\theta\right]+\frac{\sigma^2(\theta)}{E[\theta]}<\frac{1}{2}\left(E\left[\gamma\right]+\frac{\rho\sigma(\theta)\sigma(\gamma)}{E[\theta]}\right)$ and $\underline{\theta}<\frac{1}{2}\underline{\gamma}$. Note that $s^*=\frac{E[\gamma]}{2(E[\gamma]-E[\theta])}$.

after the diversification, $s^*_{ut} < s^*$ *(respectively* $s^*_{ra} < s^*$*), if and only if* $\rho > \frac{\sigma(\theta)E[\gamma]}{\sigma(\gamma)E[\theta]}$ *(respectively if* $\frac{E[\theta]}{\underline{\theta}} > \frac{E[\gamma]}{\underline{\gamma}}$*).*

Proposition 4 can be interpreted as follows. Consider the utilitarian teacher. A first necessary condition for the condition $\rho > (\sigma(\theta)E[\gamma])/(\sigma(\gamma)E[\theta])$ to hold is $\rho > 0$ and a second necessary condition is $\sigma(\theta) < 0.5\sigma(\gamma)$ (remember, that by assumption $E[\gamma]/E[\theta] > 2$). Therefore, a mean-preserving class diversification makes the teacher increase the mastery goal structure when (i) there is a strong positive correlation between ability and the mastery-performance goal index, and (ii) ability is relatively less dispersed than the mastery-performance goal index (in the sense that $\sigma(\theta) < 0.5\sigma(\gamma)$). In fact, the probability of a student of ability θ succeeding in the test is θse, which means that a complementarity exists between effort and ability in relation to achievement. Hence, being a low-ability student entails a double handicap because both the effort level, e, and the efficiency of effort, θs, are negatively affected. By the exact symmetric reasoning, there is a double advantage of being of a high-ability type. When the dispersion of abilities is high (in the sense that $\sigma(\theta) > 0.5\sigma(\gamma)$), the double inequality due to ability differences is exacerbated and it is relatively difficult to motivate the low-ability students but relatively easy to motivate the high-ability students. The utilitarian teacher therefore prefers to design a structure that is more oriented toward performance. Conversely, when the dispersion of abilities is low (in the sense that $\sigma(\theta) < 0.5\sigma(\gamma)$) and the correlation between θ and γ is positive and strong, the teacher chooses to induce effort based on students' intrinsic motivation in order to exploit the complementarity between the mastery-induced effort and ability in the student achievement function, θse.

Faced with the same mean-preserving class diversification, the Rawlsian teacher chooses to increase the mastery goal structure if the dispersion of abilities, measured by the ratio $E[\theta]/\underline{\theta}$, is larger than the dispersion of the mastery-performance goal indexes, measured by the ratio $E[\gamma]/\underline{\gamma}$. Indeed, the Rawlsian teacher only considers the student $(\underline{\theta}, \underline{\gamma})$ after the diversification and chooses a structure that is more oriented toward mastery in order to develop his intrinsic motivation and keep this student on track. The performance of the average student $(E[\theta], E[\gamma])$ decreases. This is in sharp contrast to the behavior of the utilitarian teacher, who increases the performance goal structure when ability is more dispersed than the goal index (in the sense that $\sigma(\theta) > 0.5\sigma(\gamma)$). In fact, the utilitarian teacher maximizes the average achievement by fostering the performance of *above*-average students. Note that, the performance of the average student $(E[\theta], E[\gamma])$ also decreases after the diversification.

Up until now, we have focused on how a teacher, by choosing the classroom structure, can facilitate the student's success in one test. In other words, we have dealt with the management of student motivation in the short run. However, the classroom structure also affects the way a student reacts to test results, most notably after a failure. To study the management of student motivation in the long run, we introduce a dynamic version of the model.

3 Dynamic Management of Student Motivation

We consider a twice repeated version of the static model with a representative agent presented in section 2.1.2. We assume that failure in the first period affects the student's attitude towards schooling in two ways. First, the probability of succeeding in the test in the second period decreases. This assumption echoes the cumulative nature of knowledge. Think, for example, about two successive courses where understanding the material taught in course one (math I) is a prerequisite for understanding the material taught in course two (math II). Second, we assume that a failure negatively affects the student's intrinsic motivation in the second period unless the teacher chooses a classroom structure that is sufficiently mastery-oriented in the first period.[14] Designing a mastery-oriented classroom structure in the first period can therefore be understood as a long-term investment in student motivation and as a way of maintaining students' intrinsic motivation over time, especially after a failure. Within this dynamic framework, we will see that the teacher faces a trade-off between promoting high grades in the short run through a performance structure, or allowing the student to overcome a potential failure by implementing a multiple goal structure.

3.1 The Dynamic Model

There are two periods denoted by $t = 1, 2$. To concentrate on the dynamic issues, we focus on a student with a balanced motivational pattern, $\gamma_1 = 1$. In addition, we consider the segment of ability levels for which a pure performance structure is optimal in the static framework: $\theta \geq 1/2$.

The student. In period t, the student exerts an effort e_t. We denote by x_t the random variable equal to 1 if the test in period t is successful and 0 otherwise.

The teacher. She chooses a classroom structure $s_t \in [0, 1]$. As before, classroom structures are

[14] Under a performance-oriented classroom structure, the teacher emphasizes that success is measured by good grades and proving ability, but does not emphasize that success is also measured by mastery and self improvements. The effects of the message conveyed by the structure are amplified by the test result. For simplicity, we assume that a performance message in period one affects the student's goal orientation only when the first period test is not successful. His mastery goal vanishes. There is a vast body of empirical evidence supporting the idea that, after a failure, performance-goal oriented students report more negative self-related thoughts and less interest in the learning than mastery-goal oriented students (e.g. Ames 1992; Anderman and Wolters, 2006).

substitutes, and a higher s_t means that the structure is more performance-goal-oriented, whereas a smaller s_t means that the structure is more mastery-goal-oriented. An intermediate level of s_t corresponds to a multiple goal structure. The teacher's payoff function for period t depends on the expected test result in this period, $E(x_t|h_t)$, where h_t is the history of the game at the beginning of period t.

Test result. For period one, we take

$$x_1 = \begin{cases} 1 & \text{with probability} \quad \theta s_1 e_1 \\ 0 & \text{with probability} \quad 1 - \theta s_1 e_1 \end{cases} \tag{6}$$

We assume that the probability is unchanged after a success but negatively affected after a failure. If the realized value of x_1 is equal to 1, then

$$x_2 = \begin{cases} 1 & \text{with probability} \quad \theta s_2 e_2 \\ 0 & \text{with probability} \quad 1 - \theta s_2 e_2 \end{cases} \tag{7}$$

However, if the realized value of x_1 is equal to 0, then

$$x_2 = \begin{cases} 1 & \text{with probability} \quad \theta s_2 e_2 / 2 \\ 0 & \text{with probability} \quad 1 - \theta s_2 e_2 / 2 \end{cases} \tag{8}$$

Mastery-performance goal index. We assume that the goal index is unaffected after a success: $\gamma_2(1) = 1$. Nonetheless, after a failure in period one, the goal index is affected by the classroom structure chosen by the teacher in the first period. We have:

$$\gamma_2(0) = \begin{cases} 1 & \text{if} \quad s_1 \leq \widehat{s} \\ 0 & \text{if} \quad s_1 > \widehat{s} \end{cases}$$

for a given $\widehat{s} < 1$. In other words, the student keeps his initial goal orientation after a failure if the teacher chooses a classroom structure that is sufficiently oriented toward mastery in the first period. Otherwise, the intrinsic motivation vanishes and the student becomes fully performance-oriented.

Payoffs. The student is risk-neutral. His expected payoff in period t after the history h_t can be written $E(x_t|h_t) + \gamma_t(h_t)(1 - s_t)e_t - 0.5e_t^2$. We have $h_1 = \emptyset$, $h_2 \in \{0, 1\}$, and by assumption $\gamma_1(h_1) = 1$.

We assume that the total payoff for the student is the discounted sum of his per-period payoffs. Let δ^p denote his discount factor.

The teacher's expected payoff in period t is $E(x_t|h_t)$. We assume that the total payoff for the teacher is the discounted sum of her per-period payoffs. Let δ^t denote her discount factor.

Timing of the game and strategies. In each period $t = 1, 2$,

- The teacher chooses a classroom structure $s_t \in [0, 1]$.
- The student observes s_t and exerts an effort level $e_t \in [0, 1]$.
- The teacher and the student observe the realized value of x_t.

Strategies are $s_1, s_2(1)$ $s_2(0)$ for the teacher and $e_1(s_1)$, $e_2(1, s_2)$ and $e_2(0, s_2)$ for the student.

3.2 The Subgame Perfect Equilibrium

We solve the second period (sub)game given a classroom structure s_1 and an effort e_1. We determine the student's effort and the teacher's choice of classroom structure conditional on the test result in period one.

If the student is successful in period one ($x_1 = 1$), then we have $e_2^*(1, s_2) = (\theta - 1)s_2 + 1$ and $s_2^*(1) = 1$. These results follow from expressions (3) and (4) and the assumption $\theta \geq \gamma_1/2 = 1/2$. After success in period one, the probability of the student passing the second test is sufficiently high to justify the teacher choosing a performance goal structure in period two. From Table 1, we know that, at equilibrium, the (expected) payoff for the student in period two is $U_2^{p^*}(1) = \theta^2/2$ and the (expected) payoff for the teacher is $U_2^{t^*}(1) = \theta^2$.

If the student fails the test in period one ($x_1 = 0$), then his probability of success in period two decreases. We consider two cases:

(i) $s_1 > \widehat{s}$: the teacher designed a performance goal structure in period one. In this case, the intrinsic motivation of the student totally vanishes after the failure, $\gamma_2(0) = 0$. In period two, the student chooses an effort level $e_2^*(0, s_2) = \theta s_2/2$ and the teacher chooses the classroom structure $s_2^*(0) = 1$. A performance structure is the best way to motivate a student who has lost his intrinsic interest in learning. At equilibrium, the (expected) payoff for the student in period two is $U_2^{p^*}(0) = \theta^2/8$ and the (expected) payoff for the teacher is $U_2^{t^*}(0) = \theta^2/4$.

(ii) $s_1 \leq \widehat{s}$: the teacher designed a multiple goal structure in period one in order for the student to preserve his intrinsic motivation after a failure, $\gamma_2(0) = 1$. In period two, the student chooses an effort level $e_2^*(0, s_2) = (\theta/2 - 1)s_2 + 1$ and the teacher chooses the classroom structure $s_2^*(0) = \frac{1}{2}\frac{1}{1-\theta/2}$. By establishing a multiple goal structure in period two, the teacher can build on the preserved intrinsic motivation of the student to induce effort. At equilibrium, the (expected) payoff for the student in period two is $U_2^{p^*}(0) = 1/8$. The (expected) payoff for the teacher is $U_2^{t^*}(0) = \frac{1}{4}\frac{\theta/2}{1-\theta/2}$.

We solve period one knowing $e_2^*(.)$ and $s_2^*(.)$. For a given classroom structure s_1, the student maximizes $\theta s_1 e_1 + (1 - s_1)e_1 - \frac{1}{2}e_1^2 + (\theta s_1 e_1)\delta^p U_2^{p*}(1) + (1 - \theta s_1 e_1)\delta^p U_2^{p*}(0)$. We obtain:

$$e_1^*(s_1) = \begin{cases} min\left\{1 + (\theta - 1)s_1 + (\theta s_1)\delta^p(\frac{\theta^2}{2} - \frac{\theta^2}{8}), 1\right\} & \text{if} \quad s_1 > \widehat{s} \\ min\left\{1 + (\theta - 1)s_1 + (\theta s_1)\delta^p(\frac{\theta^2}{2} - \frac{1}{8}), 1\right\} & \text{if} \quad s_1 \leq \widehat{s} \end{cases} \tag{9}$$

In period one, the student makes more effort in the dynamic model than in the static one (where effort would be equal to $1 + (\theta - 1)s_1$). The existence of a second period extends the benefits of being successful in the first period, as the student's capacity to succeed in the second test depends on his initial performance. The supplementary effort is higher, the more patient the student is.

The teacher chooses s_1^* to maximize the discounted sum of her per-period payoffs:

$$\theta s_1 e_1^*(s_1) + (\theta s_1 e_1^*(s_1)) \times \delta^t U_2^{t^*}(1) + (1 - \theta s_1 e_1^*(s_1)) \times \delta^t U_2^{t^*}(0) \tag{10}$$

where $e_1^*(s_1)$ is given by (9). Two structures are potentially optimal: $s_1^* = 1$ and $s_1^* = \widehat{s}$.
The total expected payoff for the teacher when she chooses $s_1^* = 1$ is

$$\theta(\theta + \frac{3}{8}\delta^p\theta^3)(1 + \frac{3}{4}\delta^t\theta^2) + \delta^t\frac{\theta^2}{4} \tag{11}$$

The total expected payoff when she chooses $s_1^* = \widehat{s}$ is

$$\left(\theta(\theta\widehat{s} + 1 - \widehat{s} + \theta\widehat{s}\delta^p\frac{4\theta^2 - 1}{8})\widehat{s}\right)\left(1 + \delta^t(\theta^2 - \frac{1}{4}\frac{\theta}{2 - \theta})\right) + \frac{1}{4}\delta^t\frac{\theta}{2 - \theta} \tag{12}$$

We denote by $\widetilde{s}(\theta)$ the particular value of $\widehat{s}$ that equalizes (11) and (12). The determinant of the corresponding second degree equation is

$$\Delta = 1 - 4(1 - \theta - \theta\frac{4\theta^2 - 1}{8}\delta^p)(\theta + \eta)$$

with

$$\eta = -\frac{8\delta^t(1-\theta^2)(1-\theta)^2 - 3\delta^p(4+3\delta^t\theta^2)(2-\theta)\theta^3}{32(2-\theta)+\delta^t(32\theta^2(2-\theta)-8\theta)}$$

We first consider the case where the discount factor of the student is nil, $\delta^p = 0$. Here, the determinant Δ is positive.[15] We find

$$\widetilde{s}(\theta) = \frac{1-\sqrt{\Delta}}{2(1-\theta)}$$

When $\widetilde{s}(\theta)$ is below (above) $\widehat{s}$, the structure $\widehat{s}$ yields a higher (lower) payoff for the teacher than $\widetilde{s}(\theta)$. As a result, the teacher prefers the structure $s_1 = \widehat{s}$ ($s_1 = 1$) to the structure $s_1 = 1$ ($s_1 = \widehat{s}$). Consequently, the optimal classroom structure chosen by the teacher in period one for a given ability θ and structure $\widehat{s}$ is

$$s_1^* = \begin{cases} 1 & \text{if} \quad \widehat{s} < \widetilde{s}(\theta) \\ \widehat{s} & \text{if} \quad \widehat{s} \geq \widetilde{s}(\theta) \end{cases}$$

The function $\widetilde{s}(\theta)$ is represented in Figure 5 for $\delta^t = 1$. The lower the ability, θ, the larger the area in which the teacher chooses the multiple goal structure, $\widehat{s}$, in the first period. By promoting a mix of both mastery and performance goals, the teacher accepts that the student performs less well in the first period in order for him to be able to overcome a possible failure. The choice of the multiple goal structure, $\widehat{s}$, is less appropriate for a high-ability student for two reasons, however. First, it induces a significant decrease in the expected grade in the first period compared to the situation with a performance structure. Second, the probability of passing the test is greater for a high-ability student, thereby reducing the benefit to the teacher of developing the student's failure tolerance.

Note that $\widetilde{s}(\theta)$ increases as δ^t decreases and that $\widetilde{s}(\theta) = 1$ for any θ when $\delta^t = 0$.[16] That is, as the teacher becomes less forward-looking, she is less willing to develop the student's failure tolerance at the expense of sacrificing his performance in the first period.

We now study the effect of increasing δ^p starting from zero for a given positive value of δ^t. We have

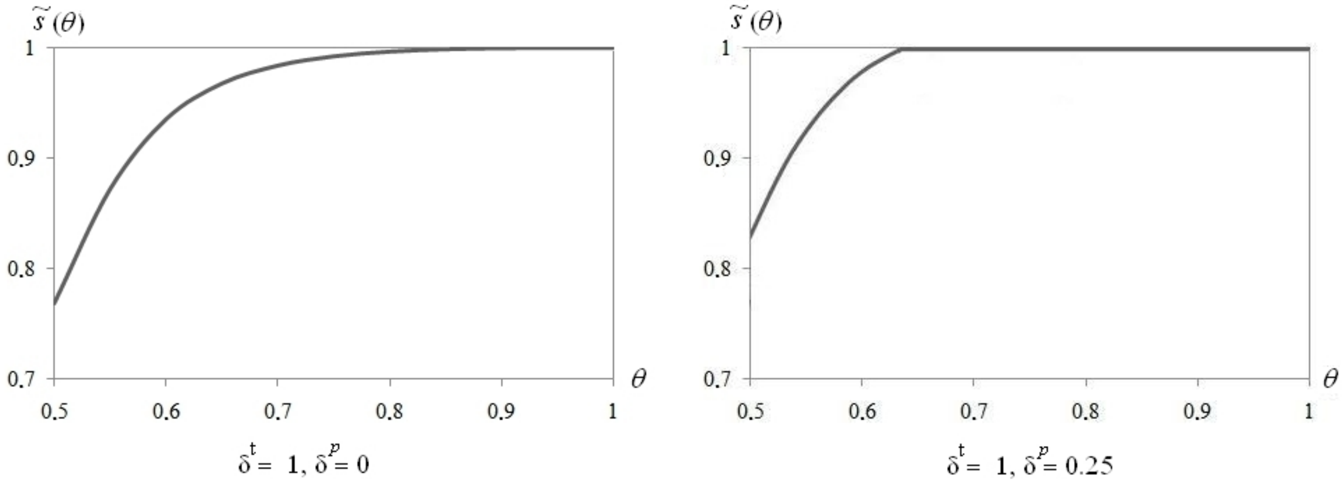
$$\widetilde{s}(\theta) = \frac{1-\sqrt{\Delta}}{2(1-\theta-\theta\delta^p\frac{4\theta^2-1}{8})}$$

It can be verified that $\widetilde{s}(\theta)$ is increasing in δ^p. A more patient student exerts a higher level of effort in period one in order to successfully enter period two. The extra effort is nevertheless smaller when the structure is more mastery-oriented in period one, because the student is then more "insured" against failure. For this reason, developing a multiple goal structure in the first period becomes less

[15] This is because $\eta < 0$ implies $4(1-\theta-\theta\frac{4\theta^2-1}{8}\delta^p)(\theta+\eta) < 4(1-\theta)(\theta) \leq 1$.

[16] When δ^t increases, η decreases. In turn, Δ increases and $\tilde{s}(\theta)$ decreases.

Figure 5: The threshold values



interesting for the teacher because it diminishes the student's incentives to exert effort in this period. It can even be verified that $\widetilde{s}(\theta)$ does not exist when δ^p is above 0.56. In this case, the teacher chooses a pure performance goal structure, which brings us back to the results of the static case. We sum up the results in the following proposition.

Proposition 5 *In a dynamic context, the teacher, if sufficiently patient, chooses a first-period goal structure that is more mastery-oriented than in the static case, if the student is not too patient. This choice of structure enables the teacher to develop the failure tolerance of the student at the expense of a short-term decrease in performance.*

The results in proposition 5 correspond to the idea developed in the achievement goal literature, which states that, by choosing a multiple goal structure, the teacher uses performance to spur efficient effort in the short run and uses mastery to increase student's failure tolerance in the long run (Ames, 1992; Barron and Harackiewicz, 2001). However, the proposition also underlines the role played by the student and the teacher's time preferences: the multiple goal structure is more effective when the teacher is sufficiently patient and the student is not too patient.

4 Policy implications

It is well established that, in many countries, education becomes more competitive on the transition from elementary to middle school (Eccles, Midgley, Wigfield, Buchanan, Reuman, Flanagan, and Mac, 1993). After entering middle school, students typically face whole class instruction and have fewer opportunities to participate in class. Teachers emphasize grades and the demonstration of ability relative to others. They attach more importance to the final achievement than to effort and progress made by the students. In general, middle-school teachers are confronted with a more demanding curriculum, tighter time constraints, and larger classes than elementary school teachers. In this perspective, a higher use of performance-oriented structures in middle school is consistent with our model (Cf. section 2.1.2, with a cost difference between structures), as it could be harder for middle school teachers, relative to elementary school teachers, to apply differentiated and individualized teaching practices. The different organizational constraints facing elementary- and middle school teachers might also suggest that teachers in middle school have a more utilitarian point of view regarding teaching practices while teachers in elementary school have a more Rawslian point of view (Cf. section 2.2, with heterogeneous students). According to Eccles, Midgley, Wigfield, Buchanan, Reuman, Flanagan, and Mac (1993), the transition to a more competitive environment may have negative consequences on students because it could result in a mismatch between students' stage of development and the school environment: evidence describes a decline in students' motivation and performance in middle school. Other authors, on the other hand, have a more positive view of the transition because they note that adolescents in middle school progressively strengthen their time preference for the future and develop a normatively based conception of ability (e.g., Harackiewicz, Baron and Elliot, 1998). In terms of our model, these changes correspond to an increase in δ^p and a decrease in γ and justify a design of a more performance-oriented classroom structure in middle school compared to primary education. Nevertheless, changes in psychological and cognitive development do not occur at the same time, the same rate, or in the same amount for all adolescents. Therefore, some students may suffer from a large mismatch between their stage of development and the school environment. To facilitate the transition from the elementary school to the middle school, it could therefore be important to design policies to decrease the cost for teachers of adopting a multiple goal structures, at least in the first year(s) of middle school.

Our model also points out that only a sufficiently patient teacher can succeed in motivating students over time. However, recent educational policies such as test-based school accountability could induce teachers to focus on students' short-term performance and prevent the building of long-term motivation. This might indicate that school accountability systems are not properly designed and do not provide the right incentives for teachers to foster student motivation. Evidence shows that under accountability pressure, teachers often utilize short-term strategies such as preemptively holding students back from taking the test (Hanushek and Raymond, 2002), e.g., by increasing the use of special education placements (Jacob, 2005). Teachers also substitute away from low-stakes subjects (Figlio, 2006), teach for the test (Jacob, 2005), and even cheat (Jacob and Levitt, 2003). In addition, accountability pressure might also induce a change in the way teachers distribute their effort among students of different abilities: teachers may prefer to neglect low-ability students and only care about students of intermediate ability. In other words, accountability systems may push teachers to adopt utilitarian preferences, as described in section 2.2. In fact, school accountability often increases the test scores of students in the middle of the achievement distribution, but not of the least academically advantaged students (Neal and Schanzenbach, 2010). Note, however, that the argument concerning a lack of long-term perspective on the part of the teacher might hinge on the use of student achievement levels as an indicator of school performance as opposed to teacher value-added. The use of adequate value-added indicators might induce teachers to become more forward-looking (Hanushek and Hoxby, 2005).

5 Conclusion

This article studies the microeconomic foundations of student motivation in schools through the lens of teaching practices. Motivation is important to understand, as it is the mechanism underlying students' effort and an influential factor in their performance. Educational psychologists have long advocated the development of multiple goals to ensure stable motivation among a wide range of students. In an article from 2008, leading researchers in educational psychology wrote:

> "*several studies have found positive effects of performance-approach goals, but not mastery goals, on grades in high school and college classes. In contrast, several studies have found positive effects of mastery goals, but not performance goals, on interest in classes. (...) Considered together, this pattern of findings supports a multiple goal perspective in which mastery and performance-approach goals can both promote important, but distinct, educational outcomes.*" (Harackiewicz, Durik, Barron, Linnenbrink, and Tauer, 2008)

On the other hand, education economists have mainly focused on the use of extrinsic motivation and the creation of competitive learning environments. This could result in educational policies being recommended under which only a small range of students are motivated to exert effort. As we have illustrated with the transition from elementary to middle school, extrinsic teaching practices can have detrimental effects when applied to an entire group of students with varying abilities and levels of interest. Our work provides a framework for clarifying how teaching practices interact with students' extrinsic and intrinsic motivation, the efficiency of their study strategies, and their achievement.

We show that, faced with high-ability students, a teacher can promote academic achievement by designing a performance goal structure. The reason is twofold. First, the effort exerted by a student under this classroom structure reflects his ability level. Second, students are more focused on the teacher's demands, so that each unit of effort exerted is more efficient. For low- or intermediate-ability students, the teacher cannot solely rely on extrinsic rewards to foster performance, but can promote better academic performances by choosing a classroom structure that is more oriented toward mastery goals (i.e., a multiple goal structure). The reason is that such a structure allows the teacher to (i) induce an effort fueled by students' interest in the task, (ii) develop students' joy of learning and therefore prevent them from adopting avoidance behavior, (iii) reduce students' risk aversion by making them less focused on the test result, and (iv) develop students' capacity to overcome failure by maintaining a persistent interest in learning over time. To sum up, a multiple goal structure yields the optimal mix between the strong, but potentially unstable, incentives corresponding to performance goals, and the more stable, but potentially less efficient, incentives corresponding to mastery goals.

Our framework also explains how teacher characteristics, such as the relative implementation cost of classroom structures, whether teachers are utilitarian or Rawlsian maximizers and how teachers discount students' future achievements, affect the choice of classroom structure. For instance, applying a mastery-oriented classroom structure is more costly relative to a performance goal structure, as the mastery goal structure requires more tailored and less standardized teaching practices. We have suggested that the cost difference between structures is higher for middle school teachers because they face a more demanding curriculum and tighter time constraints relative to primary school teachers. This could explain why middle school classroom structures are more performance-oriented than primary school structures. This could also explain student drop out as the change in school environment may be detrimental for students who have not yet developed a normatively based conception of ability or higher preferences for the future.

Our analysis suggests that, if we want teachers who are capable of triggering student motivation and achievement, it is essential to create a school environment in which (i) teachers do not face too many organizational constraints, such as class size and a too demanding curriculum, that prevent them from choosing the appropriate classroom structure, (ii) teachers are able to view student motivation and achievement as a long-term process (e.g., accountability systems need to be designed to make teachers more forward looking), and (iii) the objectives as regards which students are to be prioritized must be chosen carefully (e.g., Utilitarian or Rawslian objectives).

Whether a greater emphasis on classroom policies and teaching practices are more effective in boosting student motivation, effort and performance than current governance reforms, such as quasi-markets and school accountability, is a question that requires empirical study.

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